Majorana edge states in interacting one-dimensional systems

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We show that one-dimensional electron systems in proximity of a superconductor that support Majorana edge states are extremely susceptible to electron-electron interactions. Strong interactions generically destroy the induced superconducting gap that stabilizes the Majorana edge states. For weak interactions, the renormalization of the gap is nonuniversal and allows for a regime, in which the Majorana edge states persist. We present strategies how this regime can be reached.

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Introduction. The possibility of realizing Majorana bound states at the ends of one-dimensional (1D) conductors formed by topological insulator edge states, semiconductor nanowires or carbon nanotubes in the proximity of a superconductor [1–8], as well as by quasi-one-dimensional superconductors [9] has led recently to much activity. An important factor for the interest is the potential application of the Majorana edge states as elementary components of a topological quantum computer [7, 10–12]. In a nanowire the Majorana edge modes exist because of the p-wave nature of the induced superconductivity, which is the result of the projection of the superconducting order parameter onto the band structure of the wire, consisting of helical, i.e., spin (or Kramers doublet) filtered left and right moving conducting modes. In such a setup, the Majorana edge states appear as particle-hole symmetric Andreev bound states at both ends of the wire, with a localization length ξ inversely proportional to the induced superconducting gap Δ, and their wave function overlap is typically proportional to $\exp(-L/\xi)$ with L the wire length. The independence and the particle-hole symmetry of the two bound states is only guaranteed if this overlap is vanishingly small, therefore large L and Δ are required.

Electron-electron interactions renormalize the properties of one-dimensional conductor and so modify Δ as well as the localization length of the bound states. In this paper, we study these interaction effects in systems with helical conduction states that are in contact with a superconductor. We show that superconductivity and Majorana edge states are stable only at weak interactions. Strong and long-ranged interactions generically suppress superconductivity and so delocalize and suppress the Majorana edge states. For weaker and screened interactions, superconductivity and the Majorana edge states remain stable only if the renormalization flow reaches the strong coupling limit for the induced superconducting gap at a correlation length $\xi \ll L$. This regime is reached for a large induced superconducting gap, best possible screened interactions, and the longest possible wire length L, which outlines the necessary strategy in the experimental search for Majorana edge states. Under these conditions, although the electron interactions in most cases substantially reduce the size of the gap, the Majorana edge states remain strongly localized at each end.

In the following, we first illustrate the effect of electron interactions on the Majorana bound states using the fermion chain model of Ref. [10]. In particular, we show that for strong interactions the gap can entirely close and the system becomes equivalent to a gapless free electron gas. Motivated by this insight, we turn to a continuum theory for the nanowires, allowing us to include the interactions more effectively and to move beyond the restriction to a half-filled chain.

Fermionic chain. The prototype model for Majorana edge states is a one-dimensional open lattice of sites $i = 1, \dots, N$ described by the model [10, 13]

$$H = -\sum_{i=1}^{N-1} \left[t c_i^\dagger c_{i+1} + \Delta c_i^\dagger c_{i+1}^\dagger + \text{h.c.} \right] - \mu \sum_{i=1}^{N} n_i, \quad (1)$$

where c_i are tight-binding operators of spinless fermions, for example the electron operators of the helical conduction bands, $t > 0$ is the hopping integral, $\Delta > 0$ the triplet superconducting gap, μ the chemical potential, and $n_i = c_i^\dagger c_i$. In terms of the Majorana fermion basis [14] $\gamma_i^1 = c_i + c_i^\dagger$ and $\gamma_i^2 = i(c_i - c_i^\dagger)$, the model is rewritten as $H = -i \sum_{i=1}^{N-1} \left[w_+ \gamma_i^2 \gamma_{i+1}^1 - w_- \gamma_i^1 \gamma_{i+1}^2 \right] - i\frac{\mu}{2} \sum_{i=1}^{N} \gamma_i^2 \gamma_i^1$, with $w_\pm = (t \pm \Delta)/2$. At $t = \Delta$ and $\mu = 0$, the only nonzero interaction is w_+, and the ground state corresponds to pairing of Majorana fermions between neighboring sites $\gamma_i^2 \gamma_{i+1}^1$, with an excitation gap of $2w_+$. In the open chain, γ_1^1 and γ_N^2 no longer appear in H and remain unpaired. They form the two Majorana bound states that are localized on a single lattice site at each edge of the wire and can be occupied at no energy cost. For $\mu \neq 0$ or $\Delta \neq t$, the two edge Majorana modes are coupled to the bulk system and their spatial extension becomes larger, on the order of $\xi \sim a/\ln|w_+/w|$, with $w = \max\{|\mu|, |w_-|\}$ and a the lattice constant. In the finite system, the overlap of the two Majorana states at both ends of the chain is proportional to $e^{-Na/\xi}$, and the two states are independent only for $Na \gg \xi$.

In such a system, interactions between the fermions critically affect the existence and stability of the Majorana edge states. Indeed, they lead not only to a further coupling of the Majorana edge states to the bulk system, but also can substantially reduce the bulk gap size. As an illustration, we include into the model the repulsive nearest neighbor interaction $H' = U\sum_{i=1}^{N-1}(n_i - 1/2)(n_{i+1} - 1/2)$, with $U > 0$. The mean field contribution of this interaction is inessential. The direct part can be removed by further tuning μ as well as the edge potentials, while the exchange part adds to $w_\pm$ and the w_- contribution can be removed by tuning $t - \Delta$. Quantum fluctuations, however, cannot be suppressed, and their role reaches much further.

Indeed, it is straightforward to show that interactions can entirely close the superconducting gap. For strongly interacting $t = \Delta = U/4$ we can map H by a Jordan-Wigner transformation to the spin chain $H = t\sum_{i=1}^{N-1}(\sigma_i^x\sigma_{i+1}^x + \sigma_i^z\sigma_{i+1}^z)$, where $\sigma_i^{x,y,z}$ are spin 1/2 operators (normalized to ± 1) defined by $c_i = \frac{1}{2}(\sigma_i^x + i\sigma_i^y)\prod_{j<i}\sigma_j^z$. By a further Jordan-Wigner transformation to new fermion operators $\tilde{c}_i = \frac{1}{2}(\sigma_i^z + i\sigma_i^x)\prod_{j<i}\sigma_j^y$ we then see that $H = -2t\sum_{i=1}^{N-1}(\tilde{c}_i^\dagger\tilde{c}_{i+1} + \tilde{c}_{i+1}^\dagger\tilde{c}_i)$, which describes a free gapless fermion gas in which the localized states have disappeared.

The role of interactions are therefore crucial for understanding the stability and existence of the Majorana edge states. In the following we use a continuum description for a quantitative analysis, which allows us to include the interactions more effectively, first at half filling as in the discrete model, then away from half filling.

Continuum model. For the continuum theory, we focus on a quantum wire with Rashba spin-orbit interaction in a magnetic field with proximity induced singlet superconductivity [3–6]. The non-interacting part of the Hamiltonian for the quantum wire can be written as a sum of two parts, $H_0 = H_0^{(1)} + H_0^{(2)}$, where $H_0^{(1)}$ is given by (throughout the paper $\hbar = 1$)

$$H_0^{(1)} = \int dr \Psi_\alpha^\dagger \left[\left(\frac{p^2}{2m} - \mu\right)\delta_{\alpha\beta} + \alpha_R p\, \sigma_{\alpha\beta}^x - \Delta_Z \sigma_{\alpha\beta}^z\right]\Psi_\beta, \tag{2}$$

where Ψ_α is the electron operator for spin α, the summation over repeated spin indices, α, β, is assumed, r is the coordinate along the wire, $p = -i\partial_r$, α_R is the spin-orbit velocity, and Δ_Z is the Zeeman energy of the magnetic field applied along the spin z direction perpendicular to the spin-orbit selected spin x direction. The second part, $H_0^{(2)}$, includes the induced singlet superconducting term with order parameter Δ_S and is expressed as, $H_0^{(2)} = i\int dr \Delta_S \Psi_\alpha^\dagger \sigma_{\alpha\beta}^y \Psi_\beta^\dagger/2 +$ h.c. Without interactions, $H_0^{(1)}$ has the eigenvalues $\epsilon_\pm = p^2/2m \pm \sqrt{(\alpha_R p)^2 + (\Delta_Z/2)^2}$ and corresponding eigenmodes $\Psi_\pm(p)$. Expanding the singlet superconducting term in this eigenbasis leads to superconducting order parameters of the triplet (within Ψ_- and Ψ_+ subbands) as well as of the singlet type (mixing Ψ_- and Ψ_+ subbands). The Majorana edge states require triplet pairing [2–7, 15, 16], which is achieved by tuning the chemical potential to lie within the magnetic field gap such that only the Ψ_- subband is occupied. In Ref. [6], Majorana edge modes were derived using the full Hamiltonian $H_0^{(1)} + H_0^{(2)}$ and were shown to exist in the limit $\Delta_Z > \sqrt{\Delta_S^2 + \mu^2}$. The same physics is also obtained by restricting to the occupied Ψ_- subband, which will be assumed in the following. For $\Delta_Z \gg \Delta_S,\ \alpha_R k_F$, with $k_F \approx \sqrt{m\Delta_Z}$, the pairing then takes the compact form [2–7, 15, 16]

$$H_0^{(2)} \approx (\Delta/k_F)\int dr \Psi_-^\dagger(r) p \Psi_-^\dagger(r) + \text{h.c.}, \tag{3}$$

with the effective triplet superconducting gap $\Delta = \Delta_S(\alpha_R k_F/\Delta_Z)$.

In the following we work in the diagonal basis [17] with the fermions confined in the $r > 0$ region. The open boundary condition forces the fermion fields to vanish at the boundaries, thus $\Psi_-(r = 0) = \Psi_-(r = L) = 0$, where L is the length of the wire. In terms of the slowly varying right, $\mathcal{R}(r)$, and left, $\mathcal{L}(r)$, moving fields, the field $\Psi_-(r)$ acquires the form, $\Psi_-(r) = \sum_k \sin(kr)c_-(k) = e^{ik_F r}\mathcal{R}(r) + e^{-ik_F r}\mathcal{L}(r)$, where $c_-(k)$ is the annihilation operator in the Ψ_- subband. We note that $\mathcal{R}(r) = -\mathcal{L}(-r)$. Thus, the kinetic energy can be expressed in terms of $\mathcal{R}$ alone by $H_0^{(1)} = -iv_F\int_{-L}^{L} dr \mathcal{R}^\dagger(r)\partial_r\mathcal{R}(r)$, while the triplet-superconducting term acquires the form $H_0^{(2)} \approx -\Delta\int_{-L}^{L} dr\, \mathrm{sgn}(r)\left[\mathcal{R}^\dagger(r)\mathcal{R}^\dagger(-r) + \text{h.c.}\right]$. The noninteracting case can therefore be written as $H_0 = \int_{-L}^{L} dr\ \mathbf{R}^\dagger(r)\mathcal{H}\mathbf{R}(r)$, with

$$\mathcal{H} = \begin{pmatrix} -i\frac{v_F}{2}\partial_r & -\Delta\mathrm{sgn}(r) \\ -\Delta\mathrm{sgn}(r) & i\frac{v_F}{2}\partial_r \end{pmatrix} \tag{4}$$

and $\mathbf{R}(r) = [\mathcal{R}(r), \mathcal{R}^\dagger(-r)]^T$. Using $\mathbf{R}(r) = (e^{i3\pi/4}/\sqrt{2})\sum_\epsilon [u_\epsilon(r), v_\epsilon(r)]^T\gamma_\epsilon$, where the normalized functions $u_\epsilon(r)$ and $v_\epsilon(r)$ satisfy the eigenvalue equation $\mathcal{H}[u_\epsilon(r), v_\epsilon(r)]^T = \epsilon[u_\epsilon(r), v_\epsilon(r)]^T$, we obtain $H_0 = \sum_\epsilon \epsilon\gamma_\epsilon^\dagger\gamma_\epsilon$. For $\epsilon = 0$ there exists a localized mode at each edge. At $r = 0$ it is of the form $u_{\epsilon=0}(r) \propto e^{-2\Delta|r|/v_F}$, with $v_0(r) = iu_0(r)$. The operator corresponding to the edge mode, $\gamma_0 = \int dr u_0(r)\mathcal{R}(r)$, satisfies the Majorana condition $\gamma_0 = \gamma_0^\dagger$. Thus the Majorana edge mode obtained by combining the right and left modes is given by,

$$\Psi_{\epsilon=0}^M(r) = C\gamma_0\sin(k_F r)e^{-r/\xi}, \tag{5}$$

for $L \gg \xi$, where C is the normalization constant and $\xi = v_F/2\Delta$ the localization length. Note that in 1D the decay is purely exponential. It is interesting to note that in exact analogy with the discrete lattice model, out of the two possible Majorana states that can be constructed from the fermion field, the localized Majorana that we

obtain at one edge corresponds to the choice $\Psi + \Psi^\dagger$. Analogously, the Majorana localized at the other edge corresponds to $[\Psi - \Psi^\dagger]/i$. Moreover, similar to the edge modes in the discrete model, those obtained in the continuum limit vanish at alternate sites for half-filling. However, the result obtained in the continuum limit is valid even away from half-filling and so more general.

Interaction effects. Next we include interactions between the fermions given by $\int drdr' V(r-r')\rho(r)\rho(r')$ with $V(r)$ being the repulsive potential and $\rho(r)$ the fermion density. Interactions in general reduce Δ, and as a consequence ξ increases. To analyze this effect, we bosonize the Hamiltonian taking into consideration that the low-energy physics is described by a single species of fermions in the Ψ_- subband. Using the standard procedure [18], the bosonic Hamiltonian reads,

$$H = \int \frac{dr}{2}\Big[v_F K(\partial_r\theta)^2 + \frac{v_F}{K}(\partial_r\phi)^2 + \frac{4\Delta}{\pi a}\sin(2\sqrt{\pi}\theta) - \frac{U}{\pi^2 a}\cos(4\sqrt{\pi}\phi - 4k_F r)\Big], \quad (6)$$

where a is the lattice constant, the $\partial_r\phi$ field describes the density fluctuations and the θ is the conjugated field. The quadratic part in Eq. (6) includes repulsive interaction between the fermions ($K<1$), the sine term is due to the triplet superconducting term $H_0^{(2)}$ given in Eq. (3), and the cosine term describes umklapp scattering by $V(r)$. The umklapp terms play a role only in lattice systems but are absent in quasi-one-dimensional quantum wires fabricated on a two-dimensional electron gas. For fermions on a lattice near half-filling, $4(k_F - \pi/2a)L \ll 1$ and the oscillatory part inside the cosine term can be neglected. The interactions then lead to the renormalization of the coupling constants Δ, U, and K, which by standard renormalization group (RG) theory [18] is expressed by the RG equations

$$\frac{d\ln K}{dl} = \frac{\delta^2}{2K} - 2Ky^2, \quad (7)$$

$$\frac{d\delta}{dl} = (2 - \frac{1}{K})\delta, \quad \frac{dy}{dl} = (2-4K)y, \quad (8)$$

where $l = \ln[a/a_0]$ is the flow parameter with a_0 the initial value of the lattice constant. The dimensionless superconducting term at the length scale a is defined as $\delta(l) = 4a\Delta(l)/v_F$ and $y(l) = U(l)a/\pi v_F$. The initial values of the rescaled parameters are given by K_0, Δ_0, δ_0, U_0, and y_0. For $K<1/2$ the umklapp term is relevant and superconductivity irrelevant, leading to a Mott phase, whereas for $K>1/2$ the opposite is true and the system is superconducting. Near $K=1/2$ the low-energy physics depends critically on the relative strength of δ_0 and y_0. A large δ_0 compared to y_0 favors superconductivity over the Mott phase and vice-versa. An interesting scenario corresponds to the line of fixed points $\delta_0 = y_0$ and $K_0 = 1/2$, where the parameters remain invariant under the RG flow. Following Refs. [18, 19], we find that under a change of quantization axis the theory is described by a quadratic Hamiltonian. Therefore, similar to the discrete model with $t = \Delta = U/4$, the spectrum is gapless. The Majorana edge states are thus absent on the line of fixed points, as well as in the Mott phase.

Away from half-filling, the umklapp term in Eq. (6) becomes strongly oscillating and can be neglected, allowing us to set $y=0$ in Eq. (8). The remaining RG equations reduce to the standard Kosterlitz-Thouless (KT) equations under the change of variables $K \to 1/2\bar{K}$ and $\delta \to \bar{\delta}/\sqrt{2}$ [18]. The flow equation of $\Delta(l)$ is, $d\Delta/dl = (1-K^{-1})\Delta$, and its solution in terms of $K(l)$ is given by

$$\Delta(l) = \Delta_0 \frac{\sqrt{8[K(l)-K_0] - 4\ln[K(l)/K_0] + \delta_0^2}}{\delta_0 \exp[l]}, \quad (9)$$

where for small deviations of K from its initial value K_0, l is given by,

$$l \approx \frac{K_0}{\sqrt{\alpha}}\cot^{-1}\left[\frac{\alpha + k_0(k_0+x)}{x\sqrt{\alpha}}\right], \quad (10)$$

where $x = (K-K_0)/K_0$, $k_0 = 2K_0 - 1$, and $\alpha = \delta_0^2/2 - k_0^2$. Rather than linearizing the KT flow eqs. around the fixed point as is often done [18], the solutions given by Eqs. (9) and (10) are obtained by integrating the KT equations. Figure 1 shows Δ/Δ_0 as a function of K for $\Delta_0 = 0.05v_F/a_0$ and three different values of K_0, $K_0 = 0.5$, 0.6 and 0.8. For all the K_0's considered, Δ reduces from its initial value and acquires its minimum at $K=1$. Note that near $K=1$, Δ shows very little variation. For the strongly repulsive case, $K_0 = 0.5$, Δ is

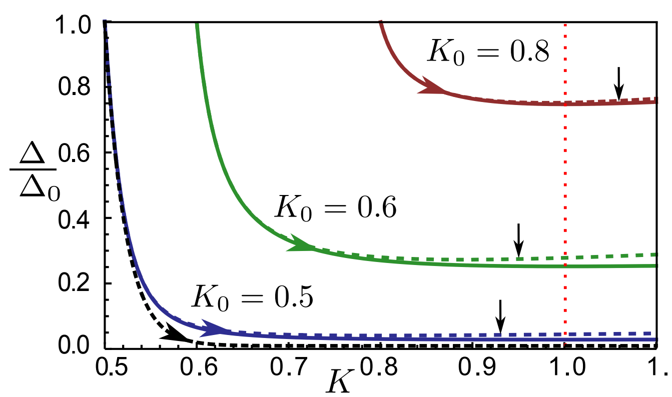

FIG. 1: RG flow of Δ/Δ_0 as a function of K for $\Delta_0 = 0.05v_F/a_0$ and the three initial values $K_0 = 0.5$, $K_0 = 0.6$, and $K_0 = 0.8$. The solid lines are obtained from the numerical integration of the KT eqs. The dashed lines are obtained from Eqs. (9) and (10) [the dashed line with the steepest decay for $K_0 = 0.5$ is obtained from Eq. (9) and $l \approx (2K_0/\delta_0^2)x$]. The flow reaches the non-interacting limit at $K=1$ (shown by the red dotted line). The vertical arrows indicate the position where $\delta = 1$ is reached.

reduced by an order of magnitude as K reaches $K \lesssim 1$. In particular, for $K \approx 0.5$ and $x \ll 1$, Eq. (10) can be approximated as $l \approx (2K_0/\delta_0^2)x$ and thus Δ has an exponential drop. More generally, the exponential decay persists as long as $x \ll \delta_0^2/(2\max\{k_0, \sqrt{|\alpha|}\})$ is satisfied. At $x \sim \delta_0^2/(2\max\{k_0, \sqrt{|\alpha|}\})$, one has to consider the full form for l as given by Eq. (10).

Next we discuss in detail the RG flow of the parameters and its consequence for the Majorana edge states. Although everywhere in the repulsive regime ($K < 1$) K has a monotonic increase and Δ a monotonic decrease, the flow can be divided into two regions based on the initial values of δ_0 and K_0. The most favorable scenario for the existence of the Majorana edge modes corresponds to the initial value (K_0, δ_0) with $K_0 > 1/2$ in the screened regime or, for $K_0 < 1/2$ with $\delta_0 > 2\sqrt{2K_0 - \ln(2K_0 e)}$ (i.e., above the separatrix). In these regions the flow is towards the strong coupling regime, and although Δ decreases monotonically it remains finite. The minimum is reached at the length scale $a(l_1)$, where $K(l_1) = 1$, beyond which point Δ increases. The RG flow crosses $K = 1$ if the length scale $a(l_1)$ is shorter than any cut-off length, i.e., $a(l_1) < \min\{L, L_T, a(l_\delta)\}$ [where l_δ is defined as $\delta(l_\delta) = 1$ and $L_T = v_F/k_BT$ is the thermal length]. We note that $K = 1$ is a special line where the interaction has scaled down to zero and the solution is obtained exactly as in the non-interacting case without further resorting to the RG. The Majorana edge state has the same form as in Eq. (5), albeit Δ is now given by the reduced value $\Delta(l_1)$. However, for preserving the Majorana property, which is of particular interest for the quantum computational use of the Majorana edge states [7, 10], the two edge states must have minimal overlap, i.e., $\chi \equiv 2\Delta(l_1)L/v_F \gg 1$. Thus the drop in Δ due to the interactions should be compensated by increasing the length of the wire by at least a factor of $\Delta_0/\Delta(l_1)$, where $\Delta(l_1)$ can be evaluated from Eqs. (9) and (10). If, however, $a(l^*) = \min\{L, L_T, a(l_\delta)\} < a(l_1)$ then the RG will be cut-off before $K = 1$ is reached. In the scenario when $K(l^*) \lesssim 1$, we note from Fig. 1 that $\Delta(l^*) \approx \Delta(l_1)$, thus we expect that the Majorana edge state will still be described by Eq. (5) with $\Delta = \Delta(l^*)$. The second regime is the unscreened regime with $K_0 < 1/2$ and $\delta_0 < 2\sqrt{2K_0 - \ln(2K_0 e)}$. Here the flow is towards the line of Luttinger-liquid fixed points, $\Delta = 0$ and $K_0 < K < 1/2$. In a realistic scenario the flow is stopped before the fixed points are reached at a length scale given by, $a(l^*) = \min\{L, L_T\}$. If $a(l^*) = L_T$, then $\Delta(l^*) < k_BT$ and thermal fluctuations overcome superconductivity. On the other hand, if the wire-length L is the cut-off, then the superconducting term is renormalized down to $\Delta(l^*) \approx \Delta_0(L/a_0)^{1-1/K_0}$. In either case the bulk spectrum remain gapless and all correlations exhibit power-law decay. Thus, the Majorana edge states which require the presence of gapped bulk modes are absent. One way to ensure a gapped phase in the bulk is to consider a larger value for δ_0. A large δ_0 will be difficult to achieve as the proximity induced gap Δ_S is further suppressed by the small ratio, $\alpha_R k_F/\Delta_Z$. Moreover, in contrast to K_0, controlling and scaling up the strength of the superconducting order parameter is nontrivial. A simpler alternative would be to apply gates on top of the wire to screen the interactions and to increase K_0 to a larger K_0' that pushes the initial point (K_0', δ_0) above the separatrix, $\delta_0 > 2\sqrt{2K_0' - \ln(2K_0' e)}$ or beyond $K_0' > 1/2$, so that the flow is towards the strong coupling regime.

Potential candidate systems for the observation of Majorana edge states are the helical conductors formed at the boundaries of topological insulators [20, 21], InAs nanowires with strong spin-orbit interaction [2, 6, 22, 23], quasi-1D unconventional superconductors [9], carbon nanotubes [8], and quantum wires with nuclear spin ordering [24]. The latter two systems may be particularly interesting because they are readily available and support helical modes without external magnetic fields.

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